2021 James S. Rickards Fall Invitational

- 1. The first five odd triangular numbers are 1, 3, 15, 21, 45. The product of these 5 numbers $(1 \times 3 \times 15 \times 21 \times 45)$ is 42,525
- 2. To find the angle of a clock's hands at any time you can use the clock angle formula which is

$$\Delta \theta = |0.5^{\circ} \times (60 \times H - 11 \times M)|.$$

In this case we are told to find the difference between the larger and smaller angles on the clock at 8:36 which means H the value in the hours place (8) and M the value in the minutes place (36) can be substituted into the equation.

$$\Delta \theta = |0.5^{\circ} \times (60 \times 8 - 11 \times 36)|.$$
$$\Delta \theta = |0.5^{\circ} \times (480 - 396)|.$$
$$\Delta \theta = |0.5^{\circ} \times (84)|.$$
$$\Delta \theta = |42^{\circ}|.$$

Now that we have the smaller angle on the clock we need to find the larger angle, and we can simply do that by subtracting our smaller angle from 360° . $360^{\circ} - 42^{\circ} = 318^{\circ}$. Now we have the larger angle as well. Now to find the difference between the larger and smaller angle we just subtract to get $318^{\circ} - 42^{\circ} = 276^{\circ}$.

- 3. To solve this question we must set up the inequality $\frac{98+68+78+72+x}{5} > 80$. Since we have the average from last time we need to make sure that the score on the 5th room makes the total average higher than last time's average which was 80. To start solving this inequality you can multiply 5 on both sides and have 98+68+78+72+x > 5*80. Then since we are trying to find the value of the 5th score, we can subtract the other rooms' scores from the total from last time and find the minimum value of the 5th score. x > 400 98 68 78 72. This gives us x > 84, and since we are trying to find the value that will make the average greater than last time's average, it will be greater than 84 and the least possible value greater than 84 is 85.
- 4. In this question we are asked to solve a summation. We are asked to start from 0 and go to 8, while solving the expression in parentheses. By substituting each value in for n you can get the sum by adding the result of each substituted n value. Substituting 0 would look like this:

$$6 + \sqrt{4^n} = 6 + \sqrt{4^0} = 6 + \sqrt{1} = 6 + 1 = 7$$

Following the same steps for each number through 8, you would get the following values, substituting 1 you would get 8, substituting 2 you would get 10, substituting 3 you would get 14, substituting 4 you would get 22, substituting 5 you would get 38, substituting 6 you would get 70, substituting 7 you would get 134, substituting 8 you would get 262. Since this question is asking for the sum of all of these values, your answer would be 7+8+10+14+22+38+70+134+262 = 565.

5. For the first card, we have eight choices for the 2 and 8 choices for the 7, giving 16 options. Once we pick the first card, the second card has to be the other number, and cannot be of the same suit, restricting to 6 options. This

means our probability is $\frac{16\cdot 6}{104(103)} = \left\lfloor \frac{12}{1339} \right\rfloor$ 6. $\begin{vmatrix} 3 & 4 & 4 & 3 \\ 15 & 15 & 12 & 16 \\ 5 & 5 & 6 & 5 \\ 3 & 3 & 4 & 3 \end{vmatrix} = \begin{vmatrix} 3 & 4 & 4 & 0 \\ 15 & 15 & 12 & 1 \\ 5 & 5 & 6 & 0 \\ 3 & 3 & 4 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 15 & 15 & 12 & 1 \\ 5 & 5 & 6 & 0 \\ 3 & 3 & 4 & 0 \end{vmatrix} = -1 * \begin{vmatrix} 15 & 12 & 1 \\ 5 & 6 & 0 \\ 3 & 4 & 0 \end{vmatrix} = -2$

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7. In order to solve this question we must find the lowest negative integer value that satisfies the inequality and the highest positive integer value that satisfies the inequality. Since we know that inequality is asking for |6x-4| < 48, we can set up two inequalities that will tell us the bounds for the least and greatest values that satisfy the equation. The two inequalities we can set up are 6x-4 < 48 and 6x-4 > -48. Solving each of the inequalities we get $x < 8\frac{2}{\pi}$

The two inequalities we can set up are 6x-4 < 48 and 6x-4 > -48. Solving each of the inequalities we get $x < 8\frac{2}{3}$ and $x > -7\frac{1}{3}$. This means that the lowest negative integer is -7 and the highest positive integer is 8. From this conclusion, we are now able to evaluate the product of the sum of the positive integers and the sum of the negative integers which is $(8 + 7 + 6 + 5 + 4 + 3 + 2 + 1) \times (-7 + -6 + -5 + -4 + -3 + -2 + -1) = \boxed{-1008}$

8.
$$\begin{vmatrix} 10 & 7 & 4 \\ 6 & 8 & 3 \\ 9 & 2 & 11 \end{vmatrix} = 10 \times \begin{vmatrix} 8 & 3 \\ 2 & 11 \end{vmatrix} - 7 \times \begin{vmatrix} 6 & 3 \\ 9 & 11 \end{vmatrix} + 4 \times \begin{vmatrix} 6 & 8 \\ 9 & 2 \end{vmatrix} = 10 \times 82 - 7 \times 39 + 4 \times -60 = \boxed{307}$$

9. Solving this question using polynomial long division you get:

 $\frac{x^5 - 3x^4 + 6x^3 - 12x^2 + 24x - 11}{x - 3} = x^4 + \frac{6x^3 - 12x^2 + 24x - 11}{x - 3} = x^4 + 6x^2 + \frac{6x^2 + 24x - 11}{x - 3} = x^4 + 6x^2 + 6x + \frac{42x - 11}{x - 3} = x^4 + 6x^2 + 6x + 42 + \frac{115}{x - 3}$ leaving us with a remainder of 115. Another way to solve this is by using the Remainder Theorem and substituting 3 for the values of x in the numerator. $3^5 - 3(3)^4 + 6(3)^3 - 12(3)^2 + 24(3) - 11 = 243 - 243 + 162 - 108 + 72 - 11 = 115$.

10. The formula for the area of a triangle given its coordinates is:

Area =
$$\left| \frac{A_x(B_y - C_y) + B_x(C_y - A_y) + C_x(A_y - B_y)}{2} \right|$$

Assuming A is the first coordinate, B is the second coordinate, and C is the third coordinate and substituting those coordinates into the formula, you should get an equation looking like this:

Area =
$$\left| \frac{3(10-2) + 7(2-3) + 10(3-10)}{2} \right|$$

Area = $\left| \frac{24 + -7 + -70}{2} \right|$
Area = $\left| \frac{-53}{2} \right|$
Area = $\left| \frac{53}{2} \text{ or } 26.5 \right|$

- 11. Here we are told that the product of x and y is 3072, so we can write that down as xy = 3072. We are also told that the difference between x and y is 16, so we can write that down as x y = 16. Now with these two written down, we can analyze the question and realize that to get $x^2 + y^2$ you have to square (x y). So we can write that as $(x y)^2 = x^2 2xy + y^2$. Since we now have this equation, we can now substitute the values we have to find the answer. $(16)^2 = x^2 2(3072) + y^2 \rightarrow 256 = x^2 6144 + y^2 \rightarrow 6400 = x^2 + y^2$. Since the question is asking for the value of $x^2 + y^2$, from what we calculated we know that value is $\boxed{6400}$.
- 12. In this question we are given the angle of the sector as 40° and the arc length of the sector as 2π . The question is asking for the area of the sector which we need the radius of the circle for. The formula for the arc length of a sector is arclength $=\frac{x^{\circ}}{360^{\circ}} \times 2\pi r$ with r being the radius and x° being the angle of the sector. Since we know the angle of the sector and the arc length we can find the radius: $2\pi = \frac{40^{\circ}}{360^{\circ}} \times 2\pi r \rightarrow 2\pi = \frac{1}{9} \times 2\pi r \rightarrow 18\pi = 2\pi r \rightarrow$ r = 9. Now that we have found the radius to be 9, we can use that in the formula for the area of a sector which is sectorarea $=\frac{x^{\circ}}{360^{\circ}} \times \pi r^2$. Substituting the radius and angle of the sector in we get: $\frac{40^{\circ}}{360^{\circ}} \times \pi 9^2 = \frac{1}{9} \times 81\pi = [9\pi]$.

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- 13. For this question you quite simply follow the defined functions. $(24\$6) = (24+6) \times (24-6) = 30 \times 18 = 540$. $(540\&12) = \frac{(540\times12)}{(540+12)} = \frac{(6480)}{(552)} = 11.7391$. Since the question says round to the nearest hundredth, the correct answer is 11.74.
- 14. For this question you can use the formula for the volume of an ellipsoid which is $V = \frac{4}{3} \times \pi abc$. In the case of this question, the semi-axes a, b, and c can be assigned as 8, 14, and 23 respectively. Solving with the formula and knowing $\pi = 3.14$ as mentioned in the question we get $V = \frac{4}{3} \times \pi \times 8 \times 14 \times 23 = \frac{4}{3} \times 2576\pi = \frac{10304\pi}{3} = \frac{32354.56}{3} = 10784.85$
- 15. $S = \sum_{n=1}^{\infty} \frac{n}{2^n} = \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} \cdots$ $2S = \frac{1}{1} + \frac{2}{2} + \frac{3}{4} + \frac{4}{8} \cdots$ $2S-S = S = \frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \cdots = \boxed{2}$
- 16. For this question you need to know the unit circle. It is extremely handy in solving this question. The values would add up as follows: $-1 + 1 + 1 + 2 (-1) + \sqrt{3} = \boxed{4 + \sqrt{3}}$
- 17. To solve this question, you must solve for x and substitute it back into another expression. Solved it should look like this:

$$21859x + 14568 = 189440$$
$$21859x = 174872$$
$$x = 8$$
$$3278(8) - 453$$
$$26224 - 453 = 25771$$

18. To solve this question you must consider the formula $S = \frac{a_1}{1-r}$. In this formula S is the sum of the infinite geometric series, a_1 is the first term and r is the common ratio. Knowing what each part of the formula means, we can start by substituting all the values into the equation.

$$S = \frac{a_1}{1-r}$$

$$S = \frac{\frac{3}{4}}{1-\frac{3}{4}}$$

$$S = \frac{\frac{3}{4}}{\frac{1}{4}}$$

$$S = \frac{3}{4} \times \frac{4}{1} = \boxed{3}$$

19. For this question you can quite simply solve using the arithmetic sum formula. That is $\frac{n(n+1)}{2}$ with n being the largest number in the sequence. Although you may use this formula, there is a 13 missing from the arithmetic sequence, meaning you must subtract 13 after you use the arithmetic sum formula. After doing that all you have to do is multiply that result by 2 and add 1. It should look like this:

$$2(1+2+3+4+5+6+7+8+9+10+11+12+14+15+16+17+18+19+20)+1 = 2(\frac{20(21)}{2}-13)+1 = \boxed{395}$$

20. Since this question is only asking for the last digit, you only need to track what number is changing in the units place. When looking at the question you notice a pattern that the numbers are consecutively decreasing by 2. Analyzing the numbers we see that the last number is 2005, so we can assume already that the answer will either end with 5 or 0. To be sure which one it is we can just multiply the units place of each number in the finite sequence and only consider the units place of the product to find the answer. Starting with 2021 and 2019: $1 \times 9 = 9; 9 \times 7 = 63; 3 \times 5 = 15; 5 \times 3 = 15; 5 \times 1 = 5; 5 \times 9 = 45; 5 \times 7 = 35; 5 \times 5 = 25$. The digit in the units place of the last product is 5, therefore the answer is 5.